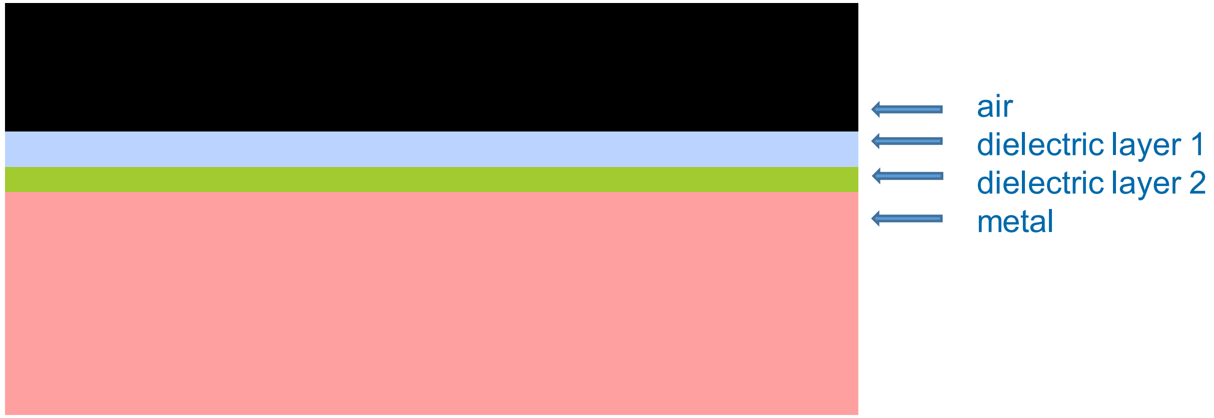
REMCOM® Electromagnetic Simulation Software

FDTD Simulation of Multilayer-Coated and Rough Surface Metals Using Surface Impedance Method

Yong Wang and Scott Langdon

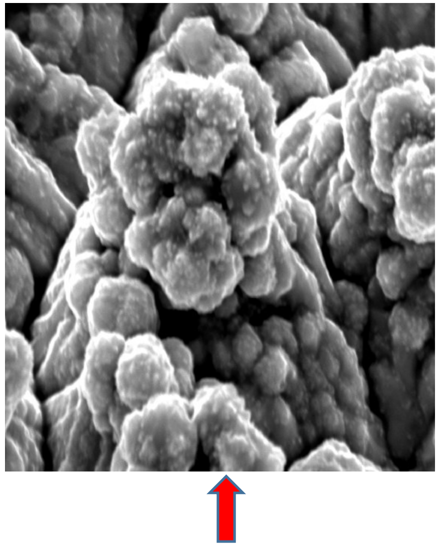
315 S. Allen St., Suite 416 | State College, PA 16801 USA | +1.814.861.1299 phone | +1.814.861.1308 fax | sales@remcom.com | www.remcom.com

The Problems to Solve and Approaches



Dielectric-coated metals

Rough surface metals



Bogatin et al, “Which one is better? Comparing Options to Describe Frequency Dependent Losses,” DesignCon, Santa Clara, USA, Jan., 2013.

Frequency dependent materials:

- Narrow band method
- Convolution-based surface impedance (SI) method
- Equivalent-circuit-based SI method

Surface Impedance of Multilayer-Coated Metals

The surface impedance of multilayer-coated metals can be expressed by:

$$Z = \frac{\omega\mu_0}{k_1}\tanh\left\{jk_1d_1 + tanh^{-1}\left[\frac{k_1}{k_2}\tanh\left[jk_2d_2 + tanh^{-1}\left[\frac{k_2}{k_3}\tanh\left[jk_3d_3 + \right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left. \ldots tanh^{-1}\left[\frac{k_{n-1}}{k_n}\tanh\left(jk_nd_n + tanh^{-1}\frac{k_n}{k_{n+1}}\right)\ldots\right]\right]\right]\right]\right\},$$

where

$k_i = \omega\sqrt{\varepsilon_i\mu_i}$ is the wavenumber of layer i, ω is the angular frequency,

ε_i is the permittivity, μ_i is the permeability, and d_i is the thickness of layer i.

The layer number from the air to the lossy metal is 0 to n+1.

Note that the surface impedance Z is frequency dependent.

King et al., Lateral Electromagnetic Waves: Theory and Applications to Communications, Geophysical Exploration, Remote Sensing, Springer-Verlag New York, 1992.

REMCOM

Hammerstad and Jensen (H & J) Model

The conductivity of rough surface metals:

$$\sigma_s = \sigma_0 / \left[1 + \frac{2}{\pi} \arctan\left(1.4 \times \left(\frac{\Delta}{\delta}\right)^2\right)(SF - 1)\right]^2,$$

where

Δ is the surface roughness RMS (root mean square average of the profile height deviations from the mean line).

δ is the skin depth of the metal: $\delta = \sqrt{\frac{1}{\pi f \mu_0 \sigma_0}}$, where f is the frequency and σ_0 is the conductivity.

The *SF* is the scale factor which is related to the added surface area from the roughness over a flat surface.

Note that the rough surface conductivity σ_s is frequency dependent.

Bogatin et al, "Which one is better? Comparing Options to Describe Frequency Dependent Losses," DesignCon, Santa Clara, USA, Jan., 2013.

Huray Model

The ratio of power loss for rough surface and flat surface is:

$$\frac{P_{rough}}{P_{flat}} = \frac{A_{Matte}}{A_{Flat}} + \frac{3}{2}\sum_{i=1}^{j}\left(\frac{N_i 4\pi a_i^2}{A_{Flat}}\right) / \left[1 + \frac{\delta}{a_i} + \frac{\delta^2}{2a_i^2}\right],$$

where

- j is the number of different-size spheres (snowballs)
- N_i and a_i are the number and radius of the spheres of the i^{th} size, respectively.
- δ is the skin depth of the metal at a particular frequency.
- A_{Matte}/A_{Flat} is the relative area of the Matte base compared to a flat surface.

This real-valued Huray model is not causal.

Huray et al, "Impact of copper surface texture on loss: a model that works", DesignCon, Santa Clara, USA, Feb., 2010.

Causal Huray Model

The causal Huray model is complex-valued and can be applied in the time domain simulation. The causal Huray model factor can be written as:

$$H_c(\omega) = 1 + \sum_{i=1}^{n} H_c^i(j\omega),$$

where $H_c^i(j\omega) = \frac{K_i}{1+\left(\frac{j2\omega}{\omega_i}\right)^{-1/2}}$, $K_i = \frac{6\pi a_i^2 N_i}{A_{hex}}$, $\omega_i = \frac{2}{a_i^2 \mu\sigma}$, n is the number of different-size spheres (snowballs), N_i and a_i are the number and radius of the spheres of the ith size, respectively, A_{hex} is the hexagonal Matte surface.

The surface impedance can be expressed by:

$$Z_{rough}(\omega) = Z_s(\omega) \cdot H_c(\omega),$$

where $Z_s(\omega) = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$.

Bracken, "A Causal Huray Model for Surface Roughness," DesignCon, Santa Clara, USA, Jan.-Feb., 2012.

Rational Models

- The surface impedance of multilayer-coated and rough surface metals can be fitted into a rational function so that the they can be simulated by the FDTD method:

$$Z = \sum_{i=1}^{N} \frac{r_i}{s - p_i} + k_0,$$

where

$s = j\omega$, N is the order number of the rational model, k_0 is the constant term, r_i and p_i are the residues and poles, respectively.

- The vector curve-fitting method is used to extract the rational model from the surface impedance.
- Unlike the smooth metal case, the rational model has to be generated for each combination of coatings or different rough surfaces.

Gustavsen and Semlyen, "Rational approximation of frequency domain responses by vector fitting," IEEE Trans. Power Del., vol. 14, no. 3, pp. 1052–1061, July 1999.

REMCOM

Convolution-Based Surface Impedance Method

Using the piecewise linear recursive convolution (PLRC) technique, the update equation for the tangential electric field on a conductor surface can be written as:

$$E_t^{n+1} = k_0\hat{n} \times H_t^{n+1/2} + \sum_{i=1}^{N} \psi_i^{n+1/2},$$

where $\hat{n}$ is the surface normal,

$$\psi_i^{n+1/2} = (\chi_i - \xi_i)\hat{n} \times H_t^{n+1/2} + \xi_i\hat{n} \times H_t^{n-1/2} + \rho_i\psi_i^{n-1/2},$$

$$\chi_i = -\frac{r_i}{p_i}\left(1 - e^{p_i\Delta t}\right), \text{ and}$$

$$\xi_i = -\frac{r_i}{p_i^2\Delta t}\left[(1 - p_i\Delta t)e^{p_i\Delta t} - 1\right], \text{ and } \rho_i = e^{p_i\Delta t}.$$

Here k_0 is the constant term, r_i and p_i are the residues and poles, respectively. They are extracted from the surface impedance. Δt is the timestep and N is the number of the order.

Note that the electric and magnetic fields are not collocated in space and have half a time step offset.

Kelley and Luebbers, "Piecewise linear recursive convolution for dispersive media using FDTD," IEEE Trans. on Antennas and Propagation, vol. 44, no. 6, 792-797, June 1996.

Synchronization in Time

To synchronize the time, the interpolation of time for the magnetic fields can be applied:

$$E_t^{n+1} = k_0\hat{n} \times (H_t^{n+\frac{3}{2}} + H_t^{n+\frac{1}{2}})/2 + \sum_{i=1}^{N} \psi_i^{n+1},$$

where $\psi_i^{n+1} = (\chi_i - \xi_i)\hat{n} \times (H_t^{n+\frac{3}{2}} + H_t^{n+\frac{1}{2}})/2 + \xi_i\hat{n} \times (H_t^{n+\frac{1}{2}} + H_t^{n-\frac{1}{2}})/2 + \rho_i\psi_i^n.$

By putting the terms with $H_t^{n+\frac{3}{2}}$ together, we have

$$E_t^{n+1} = [k_0 + \sum_{i=1}^{N}(\chi_i - \xi_i)]/2\,(\hat{n} \times H_t^{n+\frac{3}{2}}) + E_{t-}^{n+1},$$

where $E_{t-}^{n+1} = k_0\hat{n} \times H_t^{n+\frac{1}{2}}/2 + \sum_{i=1}^{N} \psi_{i-}^{n+1}$, $\psi_{i-}^{n+1} = \chi_i\hat{n} \times \frac{H_t^{n+\frac{1}{2}}}{2} + \xi_i\hat{n} \times \frac{H_t^{n-\frac{1}{2}}}{2} + \rho_i\psi_i^n$, and

$\psi_i^{n+1} = (\chi_i - \xi_i)\hat{n} \times \frac{H_t^{n+\frac{1}{2}}}{2} + \psi_{i-}^{n+1}.$

REMCOM

Synchronization in Time (cont'd)

Assuming $\hat{n} = \hat{z}$, the update equation of the magnetic field:

$$H_x^{n+3/2} = H_x^{n+1/2} + \Delta t/\mu\left(-\frac{E_{z\,j+1}^{n+1} - E_{z\,j}^{n+1}}{\Delta y} + \frac{E_{y\,k+1}^{n+1} - E_{y\,k}^{n+1}}{\Delta z}\right).$$

and assuming $E_t^{n+1} = E_{y\,k}^{n+1}$, the FDTD update equation of the magnetic field H_x can be obtained:

$$H_x^{n+3/2} = \frac{1}{1+\Delta t/\mu \Delta z}\frac{k_0 + \sum_{i=1}^{N}(\chi_i - \xi_i)}{2}\left\{H_x^{n+1/2} + \frac{\Delta t}{\mu}\left(-\frac{E_{z\,j+1}^{n+1} - E_{z\,j}^{n+1}}{\Delta y} + \frac{E_{y\,k+1}^{n+1} - E_{t-}^{n+1}}{\Delta z}\right)\right\}.$$

This is the update equation. The electric field on the surface is not needed for FDTD updates, but it may be needed for loss calculation.

The update equation with synchronization is applied at every timestep and the simulation is stable without any change to the Courant stability condition.

REMCOM

Synchronization in Space

- The extrapolation of space for the magnetic field can be applied to the original equation. Assuming $\hat{n} = \hat{z}$, we have

$$E_t^{n+1} = k_0 \hat{n} \times (3H_{t\,k}^{n+\frac{1}{2}} - H_{t\,k+1}^{n+\frac{1}{2}})/2 + \sum_{i=1}^{N} \psi_i^{n+1/2},$$

where $\psi_i^{n+1/2} = (\chi_i - \xi_i)\hat{n} \times (3H_{t\,k}^{n+\frac{1}{2}} - H_{t\,k+1}^{n+\frac{1}{2}})/2 + \xi_i \hat{n} \times (3H_{t\,k}^{n-\frac{1}{2}} - H_{t\,k+1}^{n-\frac{1}{2}})/2 + \rho_i \psi_i^{n-1/2}.$

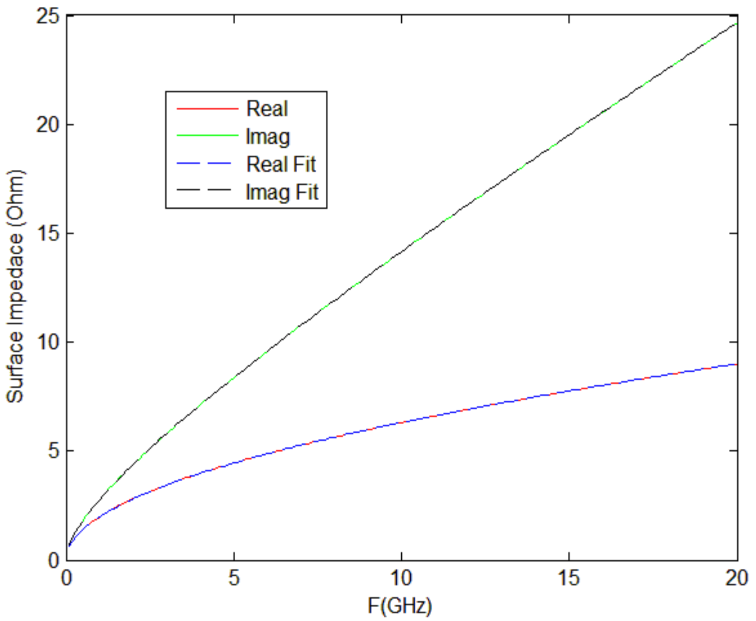
- The implementation of this scheme is straightforward. One more magnetic field and the previous timestep value above the conductor surface, such as $H_{t\,k+1}^{n\pm 1/2}$, is needed.
- Although the synchronization in space seems simple for grid aligned surface, it is difficult to apply the synchronization in space to a conformal case.

Equivalent-Circuit Based Surface Impedance Method

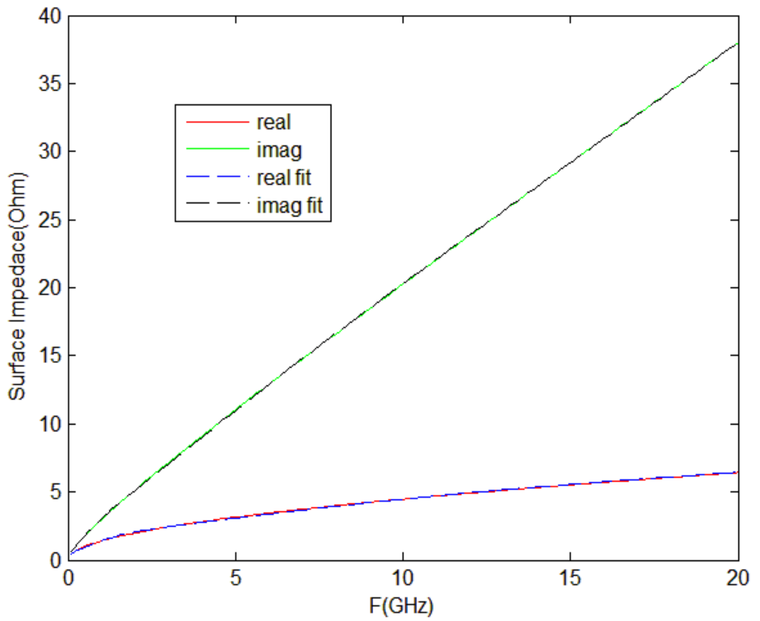
- The surface impedance of a good conductor is consistent with the input admittance of an infinite GC ladder. Assuming $\hat{n} = \hat{z}$, the GC elements is suggested to be $G = \sqrt{\frac{\omega\mu}{\sigma}\frac{\Delta y}{\Delta x}}$, C= $\sqrt{\frac{\mu}{\omega\sigma}\frac{\Delta y}{\Delta x}}$.
- If $V_x^{n+1/2} = H_x^{n+1/2}\Delta x,\ \ I_y^n = E_y^n \Delta y,\ I_z^n = E_z^n \Delta z$, the update equations for the GC ladder are $V_i^n = V_i^{n-1} + \frac{\Delta t}{C}\left(I_i^{n-1} - I_{i+1}^{n-1}\right), i = 1, \ldots K,\ I_i^n = G(V_{i-1}^n - V_i^n), i = 1, \ldots K, I_1^n = 2G\left(V_{x\ k+1/2}^{n-1/2} - V_1^n\right) \equiv I_y^n$, where we use $H_x^{n-\frac{1}{2}}(k+\frac{1}{2})$ for $V_{x\ k+1/2}^{n-1/2}$ to update I_1^n, then I_i^n and V_i^n.
- For a finite ladder, it is assumed that $I_{k+1}^{n-1} = 0$. Note that $H_x^{n-\frac{1}{2}}(k+\frac{1}{2})$ has ½ time step offset and ½ cell size offset with I_1^n.
- The advantage of equivalent circuit method is that there is no need for the signal processing to extract the poles and residues, but it can only be applied for constant conductivity case.

Taflove and Hagness, Computational Electrodynamics: The Finite-Difference Time-Domain Method, 3rd ed., Artech House, 2005.

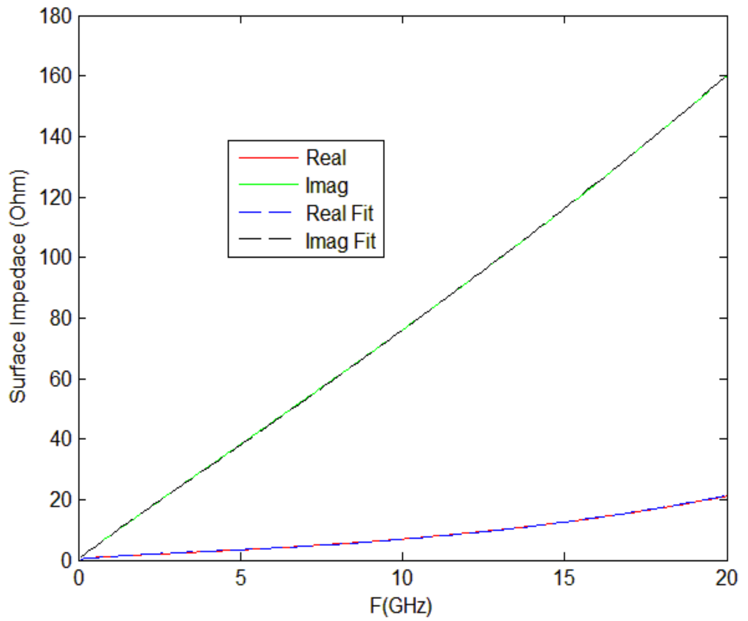
Curve-fitted Results of Multilayer-Coated Metals



Surface impedance of **one layer of normal material** (thickness d=0.1 mm, σ=2.0 S/m, ε_r=2.0) on a lossy metal (σ=1000 S/m), N=6



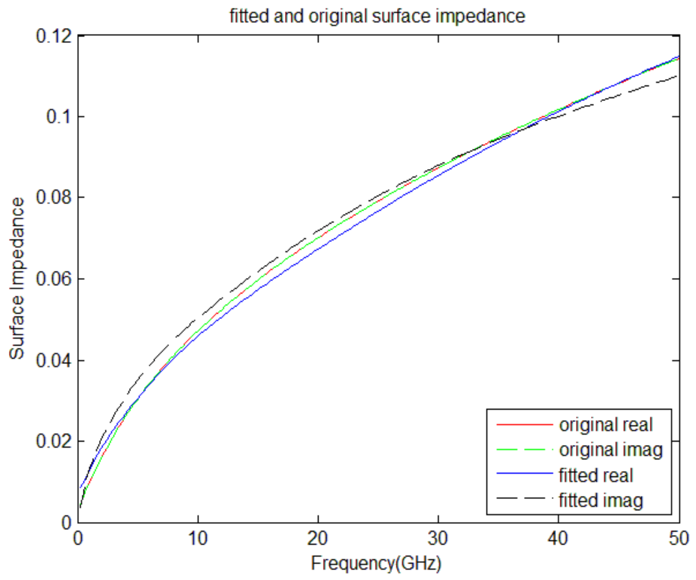
Surface impedance of **one layer of Debye material** (d=0.2 mm, static ε_s=2.0, infinity ε_{inf}=1.0, relaxation time $\tau=10^{-11}$ s) on a lossy metal (σ=2000 S/m), N=4

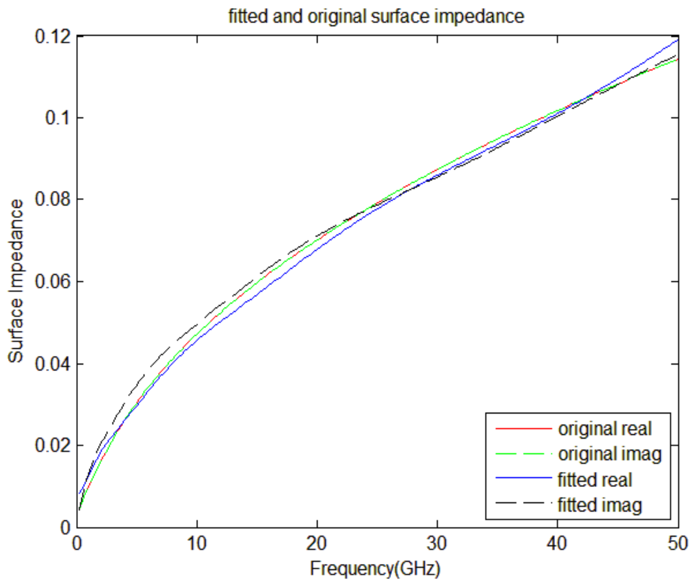


Surface impedance of **two layers of normal** (d=0.4 mm, σ=2.0 S/m, ε_r=2.0) **and Debye materials** (d=0.5 mm, ε_s=2.0, ε_{inf}=1.0, $\tau=10^{-11}$ s) on a lossy metal (σ=3000 S/m), N=4

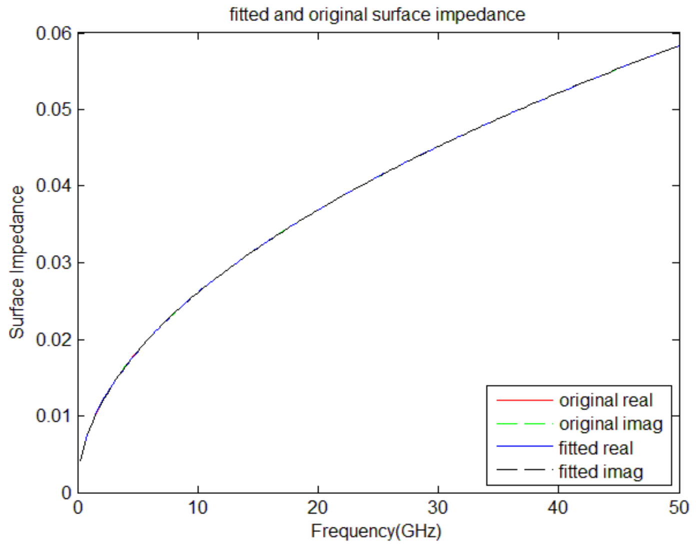
REMCOM

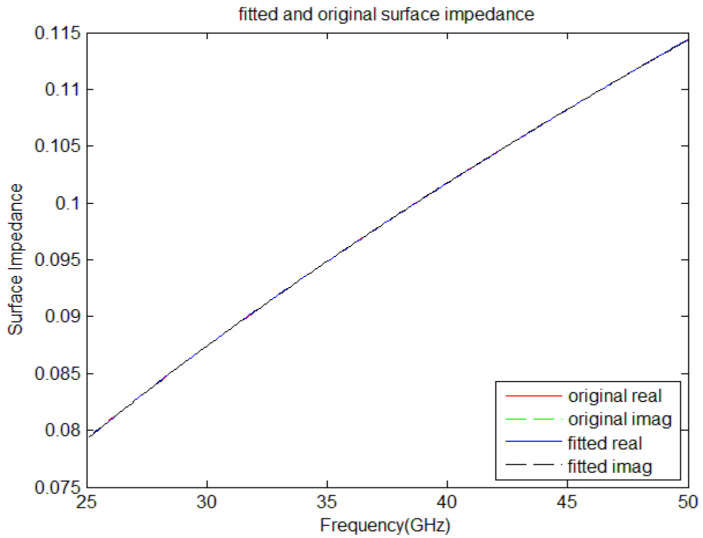
Curve-fitted Results of Rough Surface using H & J Model





The curved fitted surface impedance for **rough surface copper** (conductivity σ=5.8e7 S/m, roughness Δ=1 μm) in a broad frequency range using different orders: **6 or 10** poles.



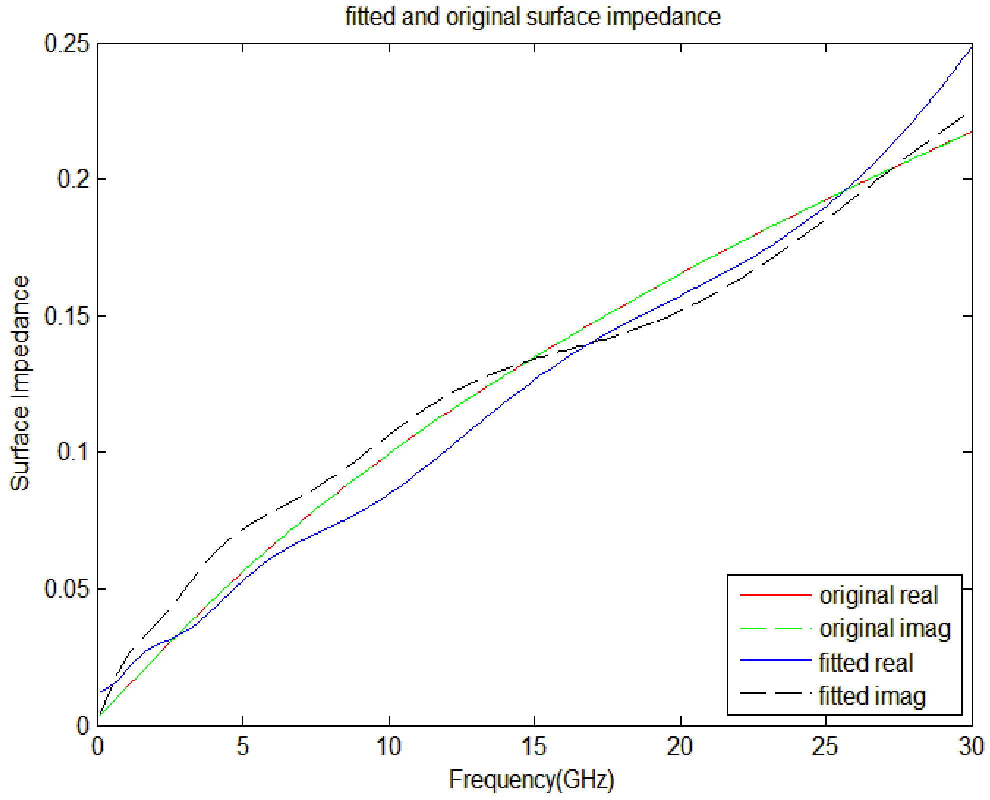


The surface impedance of the **smooth copper** can be easily fitted by using 6 poles.

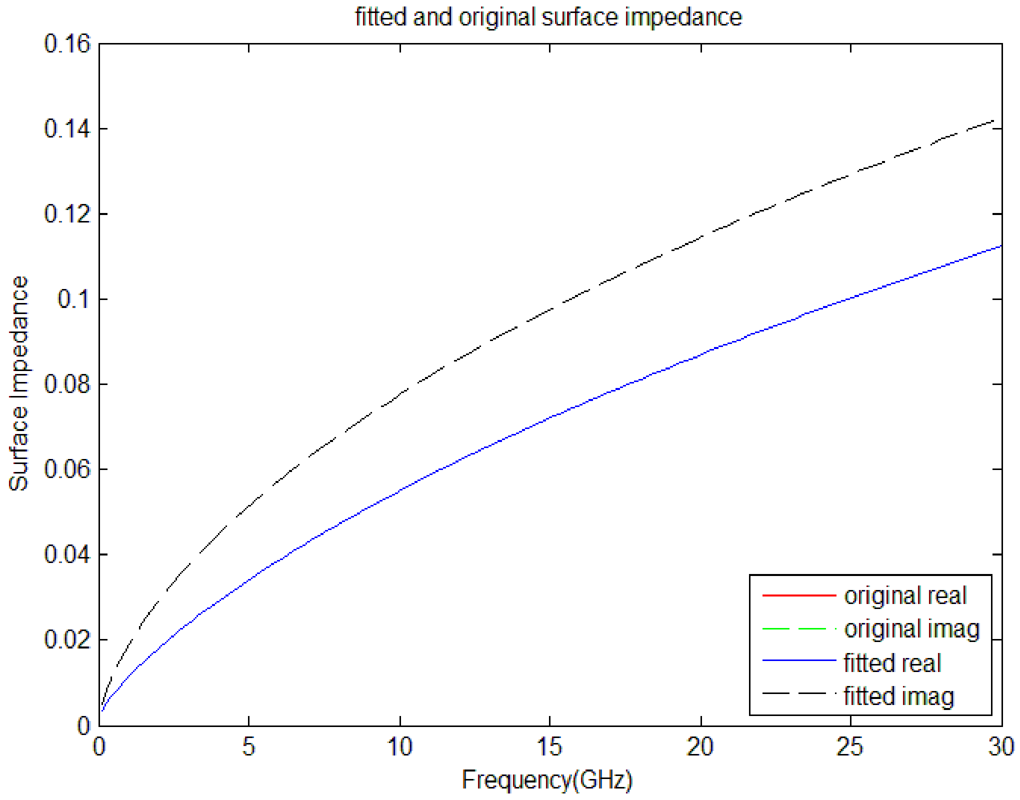
The surface impedance of the **rough surface copper** can also be well fitted for the **high frequencies**.

The impact of surface roughness is more significant at the high frequencies.

Curve-fitted Results of Rough Surface using Huray and Causal Huray Models

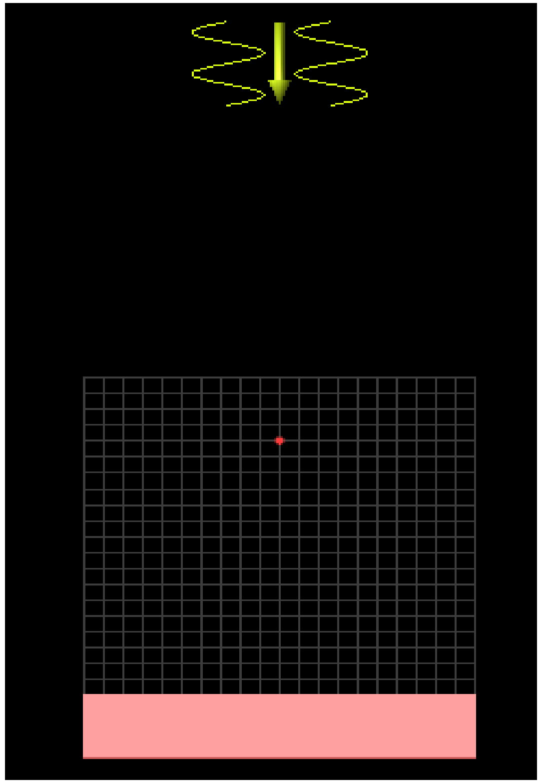


Huray model using 10th order for the rough surface copper with a_i =0.85 μm, A_{Flat} =65 μm^2, and N_i =11

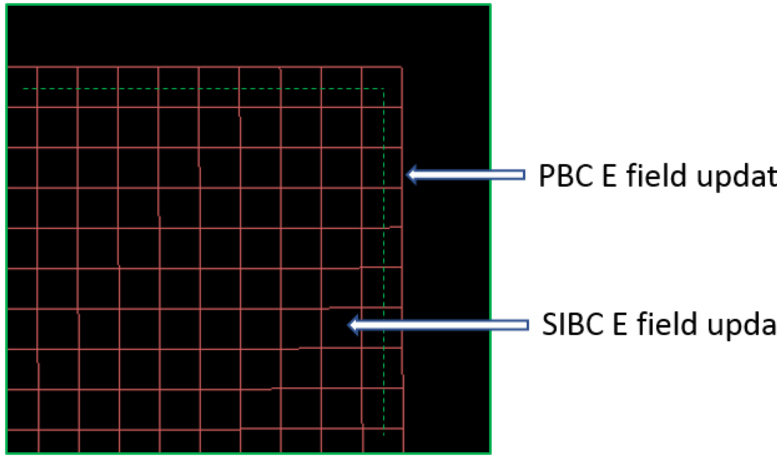


Causal Huray model using 10th order for the rough surface copper with a_i =0.85 μm, A_{hex} =65 μm^2, and N_i =11

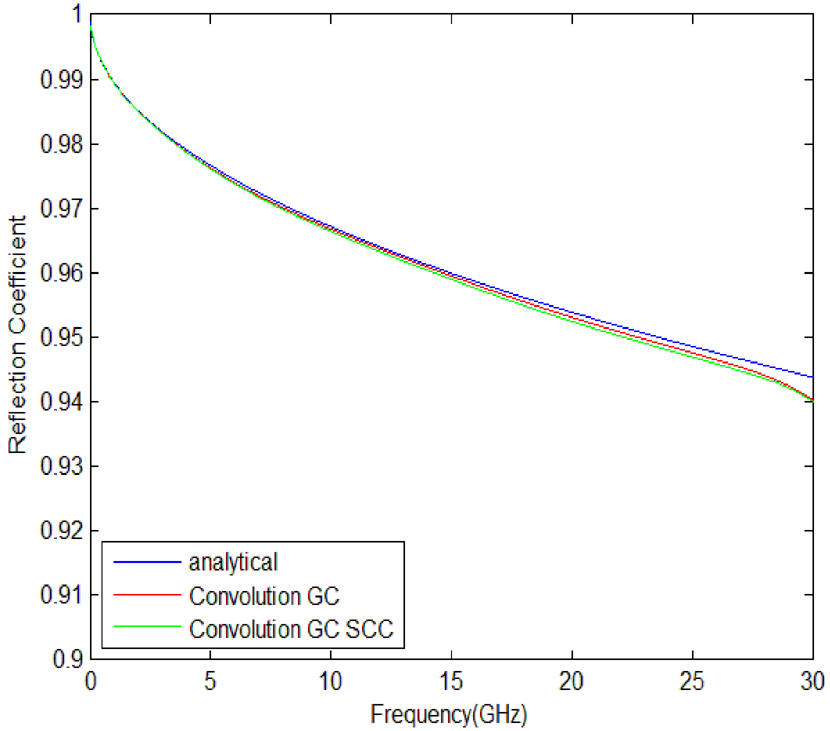
An FDTD Test Case



The reflection from coated and rough conductor with normal incidence and PBC

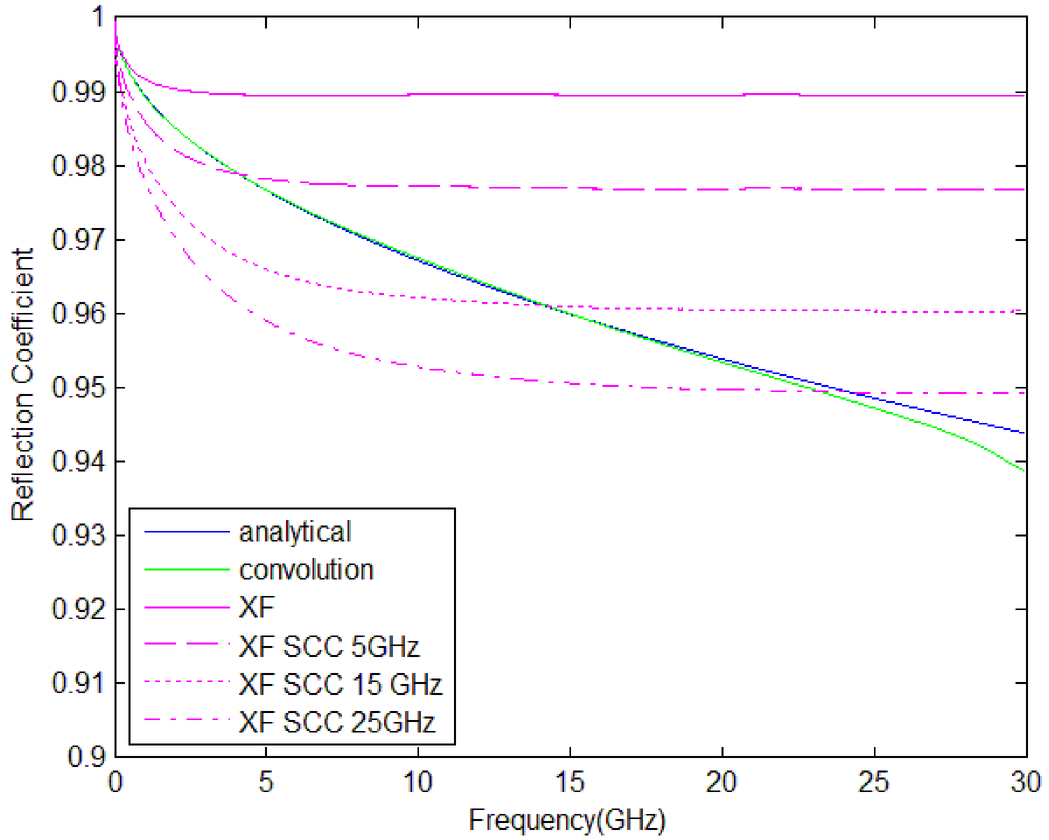


Meshes and update methods

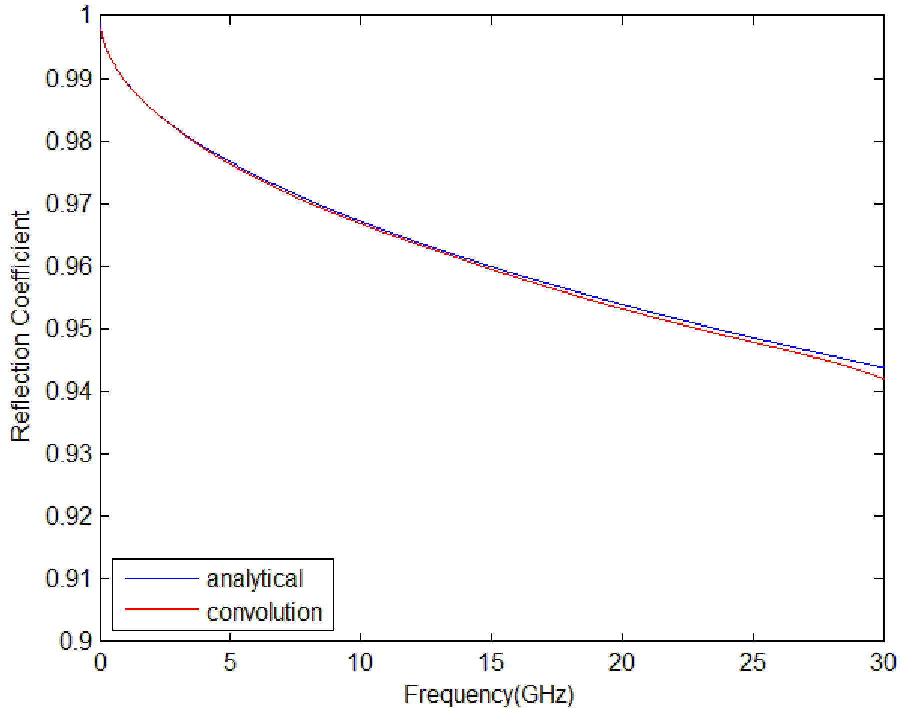


The reflection from a conductor surface

FDTD Simulation of Smooth Metals



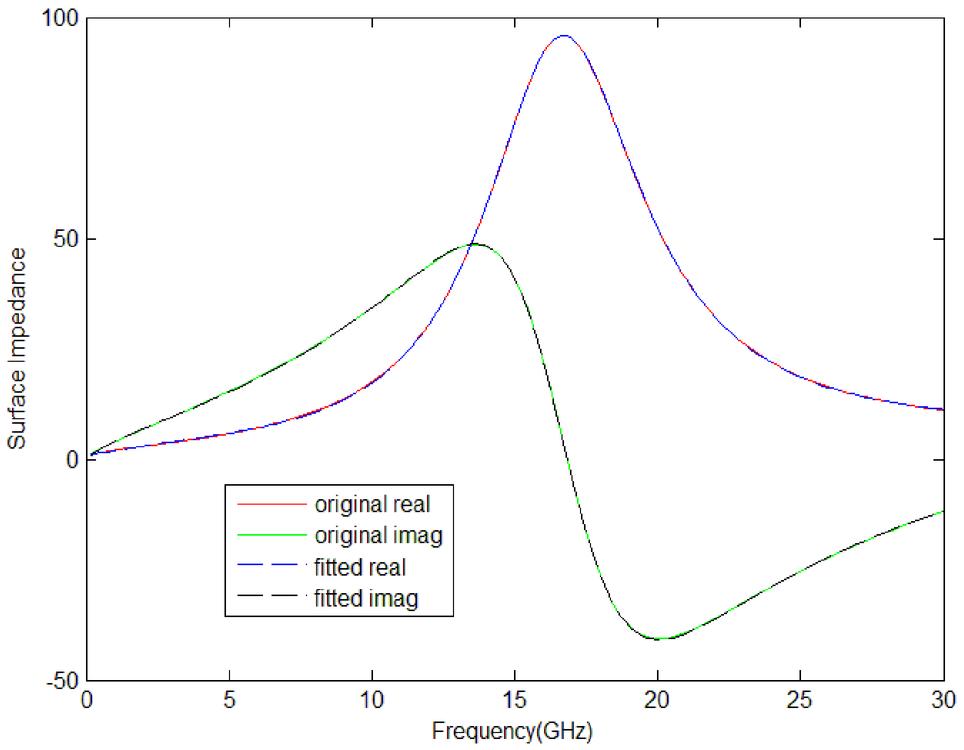
Validation of convolution method against analytical method and surface conductivity correction, σ=1000 S/m



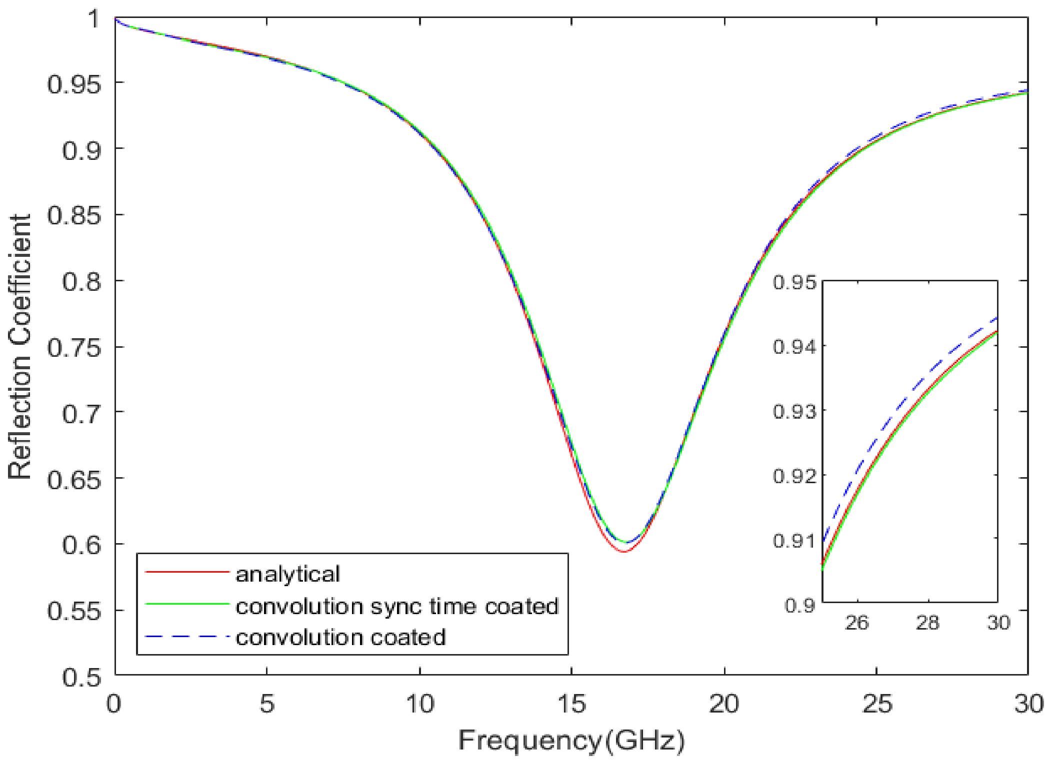
Results of convolution method using finer meshes

Chamberlin and Gordon, "Modeling of good conductors using the finite difference time domain technique", IEEE Trans. on Electromagnetic Compatibility, vol. 37, no. 2, pp. 210-216, May 1995.

FDTD Simulation of Coated Metals

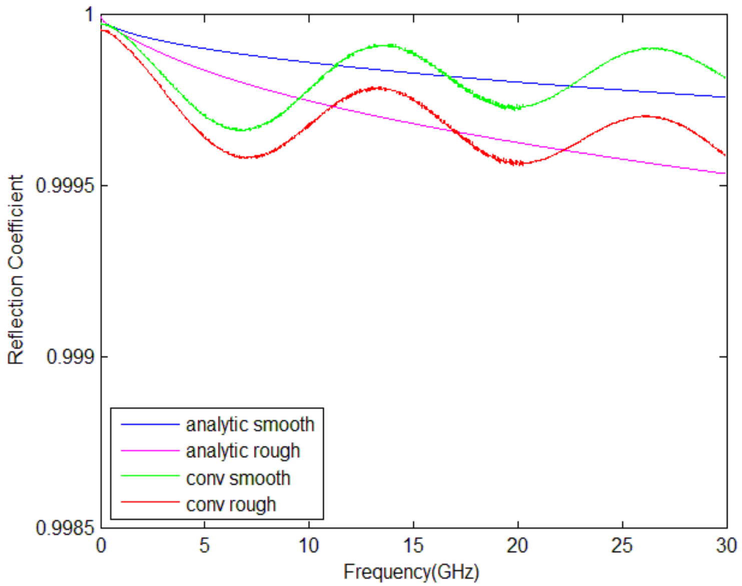


Surface impedance of a coating with d=0.25 mm, σ=0.1 S/m, ε_r=200 on a metal of σ=1000 S/m

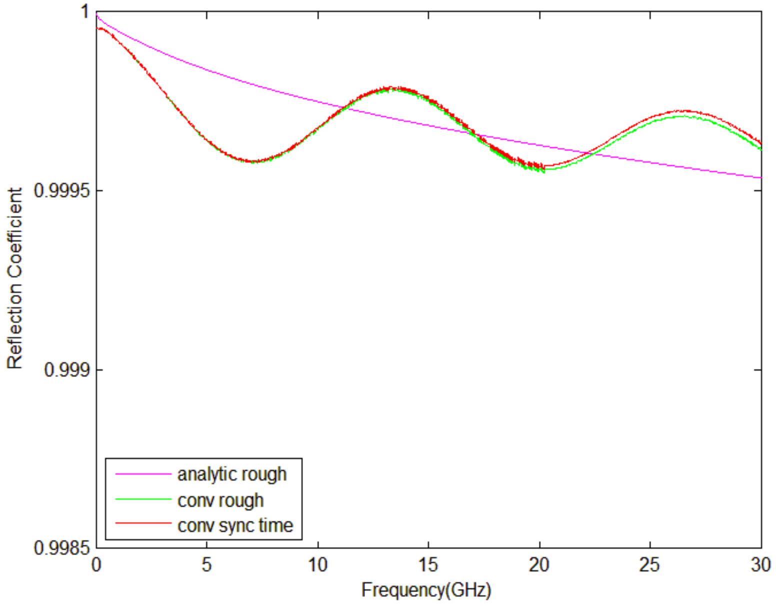


The reflection coefficients: analytical method vs. the convolution methods with and without the synchronization in time

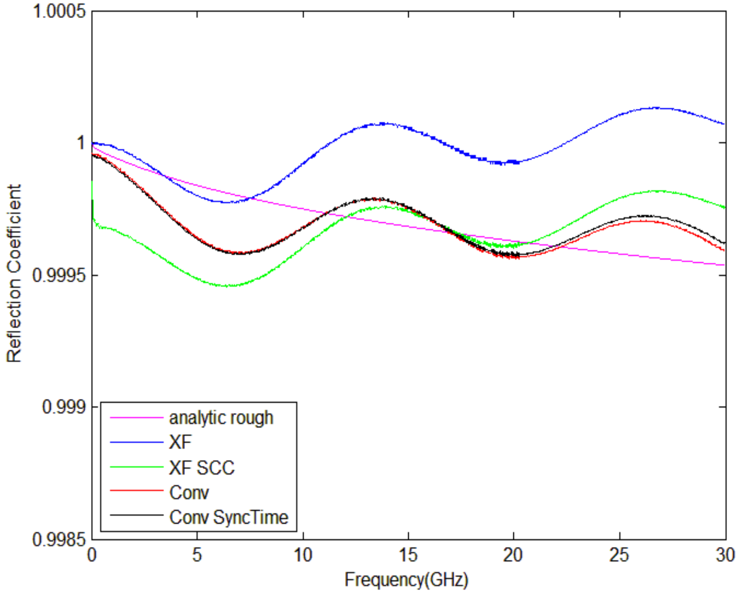
FDTD Simulation using H & J Model



Reflection coefficients from **smooth and rough** copper: analytical method vs. convolution method (σ=5.8e7 S/m, Δ=1 μm)



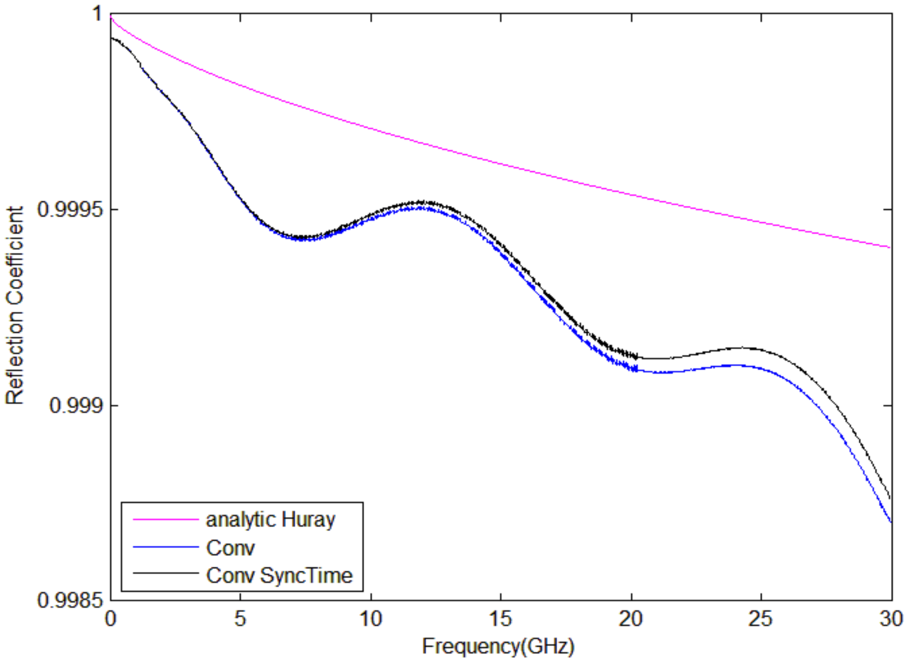
Reflection coefficients from **rough copper:** the convolution methods with/without the synchronization in time



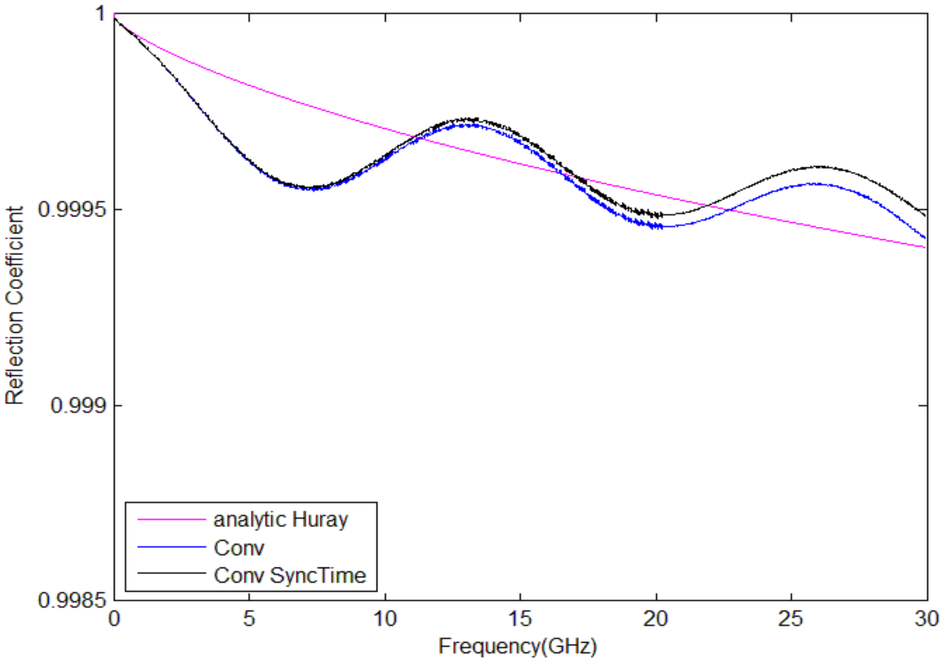
Reflection coefficients from rough surface copper using **regular update** equation with conductivity calculated at 15 GHz with/without the **surface conductivity correction**

REMCOM

FDTD Simulation using Huray/Causal Huray Models



The reflection coefficients from **Huray model** using the convolution methods with/without the synchronization in time, a_i =0.85 μm, A_{Flat} =65 μm^2 and N_i =11



The reflection coefficients from **causal Huray model** using the convolution methods with/without the synchronization in time, a_i =0.85 μm, A_{hex} =65 μm^2 and N_i =11

REMCOM

Summary

- The surface impedance of multilayer-coated and rough surface metals can be fitted by rational models if they are causal.
- The fitting process must be performed for each combination of coatings or different rough surfaces.
- The surface impedance method can be applied for FDTD simulation of multilayer-coated and rough surface metals.

REMCOM