

# FDTD Simulation of Thin Resistive Sheets

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**Abstract**— An effective approach is presented for simulation of thin resistive sheets in FDTD. The approach is based on surface impedance and piecewise linear recursive convolution technique. This approach can be combined with conformal scheme so that it can be applied to deal with arbitrarily shaped thin sheet. The simulation results for a couple of examples have shown that the approach is robust, stable and quite accurate.

## I. INTRODUCTION

An accurate modeling of thin conducting layers in FDTD was presented [1]. It was based on the surface impedance and took into account both reflection and transmission of the thin good conductor sheet. In this paper, we use the piecewise linear recursive convolution technique [2] to convert the frequency domain formulation into time domain. The implementation of the proposed approach is straightforward and it can be combined with conformal techniques to deal with arbitrarily shaped thin sheet [3]-[5]. The simulated results using the proposed approach are compared with those modeled by the infinite thin conductor scheme and the transmission coefficient method [6].

## II. PROPOSED APPROACH

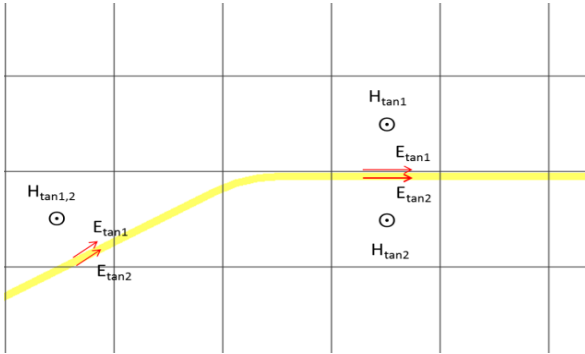


Figure 1 FDTD mesh of a thin resistive sheet

An FDTD mesh of a thin resistive sheet is illustrated in Figure 1. The tangential electric and magnetic fields in frequency domain are related by the surface impedance matrix of the thin resistive sheet. Their relationship in time domain can be expressed by

$$\vec{E}_{tan}(t) = \vec{Z}(t) \otimes \vec{H}_{tan}(t), \quad (1)$$

where  $\vec{Z}(t)$  is the surface impedance matrix. If the thin sheet is a good conductor, the  $Z_{11}$  in frequency domain can be expressed by

$$Z_{11} = \frac{1}{\sigma d} \frac{\sum_{m=0}^M \frac{s'^m}{(2m)!}}{\sum_{n=0}^N \frac{s'^n}{(2n+1)!}} = \frac{1}{\sigma d} \left( \sum_{i=0}^N \frac{r_i}{s' - p_i} + k_{11} \right), \quad (2)$$

where  $\sigma$  and  $d$  are the conductivity and thickness of the thin resistive sheet, respectively, and  $s' = -j\omega\sigma\mu d^2$  which is related to the angular frequency  $\omega$ . Using the piecewise linear recursive convolution technique, the electric field fields in the time domain can be expressed by

$$\vec{E}_{tan1}^{m+1} = \frac{1}{\sigma d} \left[ \sum_{i=0}^N (E_{tan1i11}^{m+1} - E_{tan1i12}^{m+1}) + k_{11} \vec{n} \times \vec{H}_{tan1}^{m+1} - k_{12} \vec{n} \times \vec{H}_{tan2}^{m+1} \right], \quad (3)$$

where  $E_{tan1i11}^{m+1} = (\chi_{i11} - \xi_{i11}) \vec{n} \times \vec{H}_{tan1}^{m+1} + \xi_{i11} \vec{n} \times \vec{H}_{tan1}^m + \rho_{i11} \vec{E}_{tan1i11}^m$ ,  $\chi_{i11} = -\frac{r_i}{p_i} (1 - e^{p_i \Delta t})$ ,  $\xi_{i11} = -\frac{r_i}{p_i^2 \Delta t} [(1 - e^{p_i \Delta t}) e^{p_i \Delta t} - 1]$  and  $\rho_{i11} = e^{p_i \Delta t}$ .

All other terms can be expressed accordingly. It is noted that the time step should be reduced to maintain the stability due to the cell area reduction.

Due to the nature of FDTD, the magnetic field has half a time step offset and a spatial offset in the update equation. The time averaging and other improvement techniques may be used if they do not cause stability problems. Moreover, for some conformal cells, the magnetic fields for both sides of the thin sheet may be collocated as shown in Figure 1.

### III. SIMULATION RESULTS

To test the stability and accuracy of the proposed approach, a few examples have been investigated. Here three test cases are illustrated.

#### A. Dipole near a Thin Resistive Sheet



Figure 2 A dipole near a thin resistive sheet

A thin resistive sheet is located under a dipole. The radiation efficiency of the dipole is calculated while the thin resistive sheet is modeled by the infinite thin conductor scheme, the transmission coefficient method, and the proposed approach, respectively. The resistance  $R_s$  of the thin sheet is  $270 \Omega/\square$ . The mesh size is 1 mm and the thickness of the thin resistive sheet is  $5 \times 10^{-5}$  mm. The sheet size is  $100 \times 70$  mm, the dipole length is 80 mm, and the frequency is 1.8 GHz. Table I shows the difference of radiation efficiency calculated by various approaches. It is seen that the radiation efficiency calculated by the proposed approach is close to the result calculated by the transmission coefficient method. But both results are much lower than the results obtained by using the infinite thin conductor scheme.

Table I Radiation efficiency of a dipole near a thin resistive sheet calculated by various approaches

Approach	Infinite thin	Infinite thin (surface correction)	Transmission coefficient	Proposed
Radiation efficiency (%)	99.7	95.2	46.5	45.5

#### B. Circuit with a Thin Resistive Sheet

The FDTD meshes of a circuit with a thin resistive sheet are shown in Figure 3. The resistance  $R_s$  of the thin sheet is  $270 \Omega/\square$ . The mesh size is chosen to be 1.0, 0.5, 0.25, and 0.1 mm, respectively, and the thickness of the thin resistive sheet is 0.05 mm. The simulated input impedance of the circuit is shown in Figure 4. The results show that the impedance of the thin resistive sheet simulated by the proposed approach may be affected by the mesh size. The reason for this could be the treatment of cells near the edges and the approximation due to the offset of magnetic field.

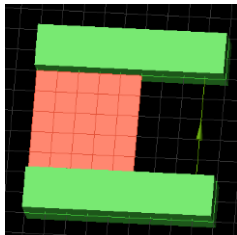


Figure 3 A circuit with a thin resistive sheet

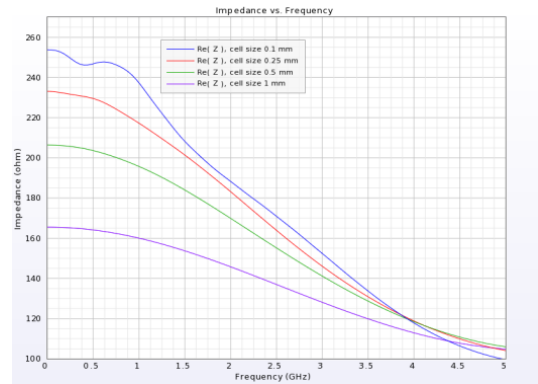


Figure 4 Simulated impedance of the circuit with a thin resistive sheet for various mesh sizes

#### C. Transmission Coefficient of Thin Conductive Sheet

The transmission coefficient of a thin conductive sheet is simulated using the total-field-scattered-field scheme and periodic boundary conditions. The thin sheet conductivity is 100 S/m and thickness is 0.1 mm. The frequency is 1 GHz. Figure 5 shows the transmission field at a point simulated by various methods. The analytical transmission coefficient is 0.347. It is seen that the result calculated by the proposed method is close to that calculated by the transmission coefficient method, and it is much better than the result obtained by the infinite thin sheet scheme.

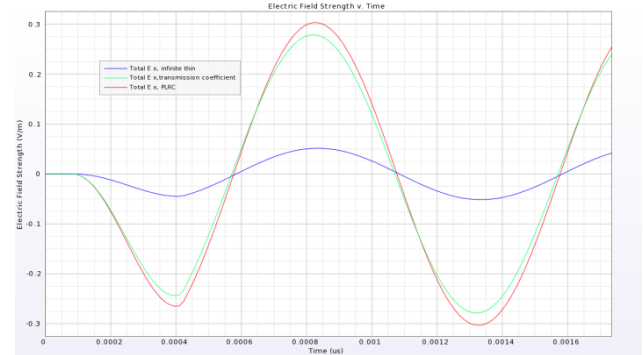


Figure 5 Transmission field vs. time at a point simulated by various methods

### IV. CONCLUSION

An effective approach based on surface impedance and piecewise linear recursive convolution technique is presented for simulating thin resistive sheets in FDTD. The simulation results showed that the approach was robust, stable and quite accurate. This approach can be applied to arbitrarily-shaped thin sheets when it is combined with conformal schemes.



#### ACKNOWLEDGEMENT

The authors would like to thank our customers in industries and colleagues at Remcom for their encouragement and assistance to this research.

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